Master 2 «Automatique & Traitement du Signal des Images»

Inverse problems and Structured Illumination Microscopy

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We address the problem of image reconstruction in structured illumination microscopy (SIM). The forward model is

$$\boldsymbol{g} = \boldsymbol{H}\boldsymbol{M}\boldsymbol{f} + \boldsymbol{n}$$

where

— $\boldsymbol{f} \in \mathbb{R}^N$ is the unknown image,

— $\boldsymbol{g} \in \mathbb{R}^N$ the collection of modulated data images,

— $\boldsymbol{n} \in \mathbb{R}^M$ the unknown noise,

— $\boldsymbol{R}$ the replication matrix, the tranpose $\boldsymbol{R}^t$ being a "sum".

— $\boldsymbol{M}$ the block diagonal modulation matrix,

— $\boldsymbol{H} = \boldsymbol{F}^\dagger \boldsymbol{\Lambda}_{\boldsymbol{H}} \boldsymbol{F}$ the block circulant convolution matrix.

The directory contains Matlab© / Octave codes to resolve the problems.

http://research.orieux.fr/teaching.html#tdsi

1 Supervised reconstruction

The naive solution (*i.e.,* the least square solution) is ill-posed. We defined the solution as the minimizer of the regularized least square solution

$$\hat{\boldsymbol{f}} \overset{\text{def}}{=} \arg\min_{\boldsymbol{f}} J(\boldsymbol{f}) = \|\boldsymbol{g} - \boldsymbol{H}\boldsymbol{M}\boldsymbol{R}\boldsymbol{f}\|^2 + \lambda\|\boldsymbol{D}\boldsymbol{f}\|^2.$$

The gradient is

$$\nabla J(\boldsymbol{f}) = 2\boldsymbol{R}^t\boldsymbol{M}^t\boldsymbol{H}^t\boldsymbol{g} + 2\boldsymbol{R}^t\boldsymbol{M}^t\boldsymbol{H}^t\boldsymbol{H}\boldsymbol{M}\boldsymbol{R}\boldsymbol{f} + 2\lambda\boldsymbol{D}^t\boldsymbol{D}\boldsymbol{f}.$$

The Hessian is

$$\nabla^2 J = 2\boldsymbol{R}^t\boldsymbol{M}^t\boldsymbol{H}^t\boldsymbol{H}\boldsymbol{M}\boldsymbol{R} + 2\lambda\boldsymbol{D}^t\boldsymbol{D}.$$

The solution is known and equals

$$\hat{\boldsymbol{f}} = \left(\boldsymbol{R}^t\boldsymbol{M}^t\boldsymbol{H}^t\boldsymbol{H}\boldsymbol{M}\boldsymbol{R} + 2\lambda\boldsymbol{D}^t\boldsymbol{D}\right)^{-1}\boldsymbol{R}^t\boldsymbol{M}^t\boldsymbol{H}^t\boldsymbol{g}$$

Unfortunately it can't be directly computed. A conjugate gradient optimization algorithm is used to reach it.

Instructions

1. Look `forward`, `forward_transpose`, and `hessian` functions and compare with equations.
2. Run `main.m` script to initialize the problem.
3. Inspect the data $\boldsymbol{g}$ (`data`), the modulation $\boldsymbol{M}$ (`grid`), the optical transfer function $\boldsymbol{H}$ (`otf`).
4. Run `main_supervised.m` to compute $\hat{\boldsymbol{f}}$.
5. Try different hyper-parameter values for `hypers(2)`.
6. Try different number of iterations.
7. Try different modulation $\boldsymbol{M}$ (in number, orientation, angles, ...).
8. Inspect the criterion values (look for the `conj_grad` function help).

2 Unsupervised reconstruction

We want to estimate the regularization parameters. Within a Bayesian approach, the posterior law writes

$$p(\boldsymbol{f}, \gamma_{\boldsymbol{f}}, \gamma_{\boldsymbol{n}} \mid \boldsymbol{g}) \propto \gamma_{\boldsymbol{n}}^{\frac{N}{2}-1} \gamma_{\boldsymbol{f}}^{\frac{N-1}{2}-1} \exp\Big(-\frac{\gamma_{\boldsymbol{n}}}{2}\|\boldsymbol{g} - \boldsymbol{HMRf}\|^2 + \frac{\gamma_{\boldsymbol{f}}}{2}\|\boldsymbol{Df}\|^2\Big)$$

where $\lambda = \gamma_{\boldsymbol{f}}/\gamma_{\boldsymbol{n}}$ in the supervised framework. With the Gibbs sampler

1. $\boldsymbol{x}^{(k)} \sim p\left(\boldsymbol{f} \mid \gamma_{\boldsymbol{f}}^{(k-1)}, \gamma_{\boldsymbol{n}}^{(k-1)}, \boldsymbol{g}\right)$,
2. $\gamma_{\boldsymbol{f}}^{(k)} \sim p\left(\gamma_{\boldsymbol{f}} \mid \boldsymbol{f}^{(k)}, \gamma_{\boldsymbol{n}}^{(k-1)}, \boldsymbol{g}\right)$,
3. $\gamma_{\boldsymbol{n}}^{(k)} \sim p\left(\gamma_{\boldsymbol{n}} \mid \boldsymbol{f}^{(k)}, \gamma_{\boldsymbol{f}}^{(k)}, \boldsymbol{g}\right)$,
4. $k \leftarrow k+1$ and return in 1,

to simulate samples, the posterior mean estimator is approximated as

$$\hat{\boldsymbol{x}} \approx \frac{1}{K}\sum_{k=0}^{K-1} \boldsymbol{x}^{(k)}.$$

Instructions

1. Run the `main_unsupervised.m` script.
2. Inspect the function and identify each part of the Gibbs sampler.
3. Identify the equation and conditional posterior law.
4. After running the script, look at the image and the posterior standard deviation.
5. Look at the hyper-parameters chains for the noise $\boldsymbol{n}$ and the image $\boldsymbol{f}$.
6. Identify the burn-in period in chains.
7. Compute the posterior mean of hyper parameters given the chains and compare the value you found with supervised algorithm in item 5.
8. Estimate the posterior standard deviation of hyper-parameters from their chains.
9. Try different values for the number of iterations and burn-in period.